TPG and PGS Thermal Conductivity

June 09, 2003

Summary

As part of the continuing study of the thermal conductivity of TPG and PGS, this report discusses two updates. Measurements of the heat flow were taken when the sample temperature was up to 60 degrees higher than previous measurements. Thus, the thermal conductivity of PGS was measured for temperatures up to –20°C. The second update is how the data is calculated. The previous data was analyzed with the assumption that the change in heat flow though any sample cross section for the sample's entire length is equal to the power change of the source heater. For the data presented in this report, it is assumed that the heat flow from the source heater through the sample is affected by additional heat loss by radiation coupling. All data from the entire study is recalculated using the fraction of power coming from the source heater through the length of the sample. The data shows that the thermal conductivity of TPG is affected by cold temperature while the thermal conductivity of PGS remains unchanged.

Increasing the sample temperature

To measure the PGS conductivity at temperatures nearer to room temperature, a small piece of copper tape (0.015" thick, 3/8" wide, 0.56" long) was inserted between the right TPG section and the LN2 heat sink (Figure 1)[1]. The temperature of the RTD 201 is more sensitive to the change in heat flow through the sample because of the increase of thermal resistance between the RTD 201 and the LN2 sink. A second heater H2 is added to the test setup in order to modify the temperature of the sample on its cold side.

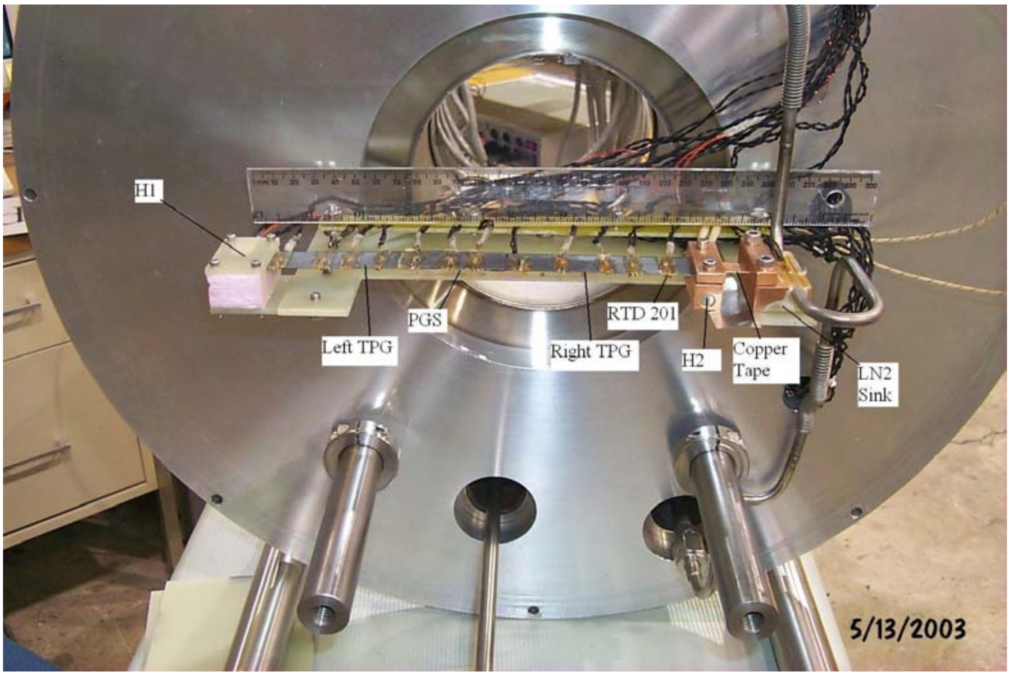

Figure 1- Copper tape is installed on the cold side of the TPG_PGS sample to increase the sample temperature. The power steps to measure the conductivity are produced by the heater H1. Heater H2 modifies the temperature of the sample's cold side.

[1] The apparatus described in the BTeV document 1650 and in the report "TPG and PGS Thermal Conductivity with no magnetic field applied. March 31, 2003" has the sample cold side coupled directly with the LN2 heat sink. The highest PGS temperature allowed by this setup is about -80 C.

Heater power fraction reaching the sample cold side
Figure 2 shows that the RTD201 temperature does not increases linearly with the heater H1 power. Table 1 shows the data that was used in Figure 2.

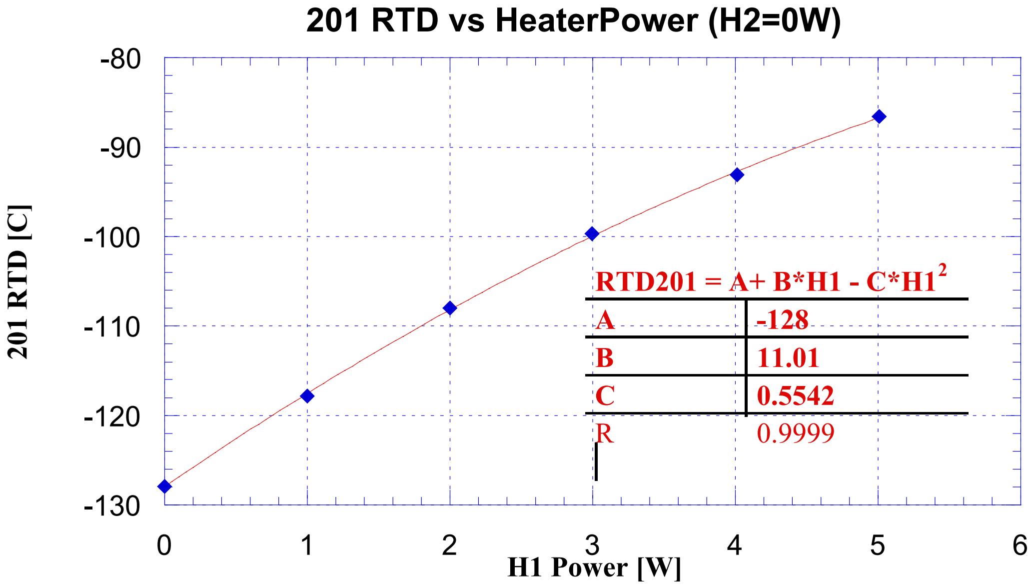

Figure 2- RTD 201 temperature versus the H1 heater power. The H2 heater is off.

Table 1 – RTD201 Temperatures for Increasing H1 Power

H1 Power	RTD201
W	C
0	-127.9
1.001	-117.8
2.003	-108
2.995	-99.7
4.013	-93.1
5.009	-86.6

The copper thermal conductivity in this temperature range is 400 - 450 W/mK. It depends on the copper tape chemical and physical purity. It also slightly depends on the temperature (the copper Debye temperature is 310K)[2]. The thermal resistance of this copper tape is about 9 K/W. The thermal resistance of the two copper-copper contacts and of the TPG-copper contact must be added to the copper tape resistance to have the total thermal resistance between the RTD 201 and the LN2. Neglecting the temperature dependence of this total thermal resistance in the temperature range of our data, we will use the non-linear growth of the temperature in Figure 2 to evaluate the fraction of the H1 power steps reaching the sample cold end.

Figure 3 shows a schematic of the heat flow through the sample. This schematic is used create a mathematical model of the heat flow from the heater H1 through the length of the sample to RTD201.

[2] G.K. White. Experimental Techniques in Low-Temperature Physics. Oxford Science Publications. Pag 297 of the third edition.

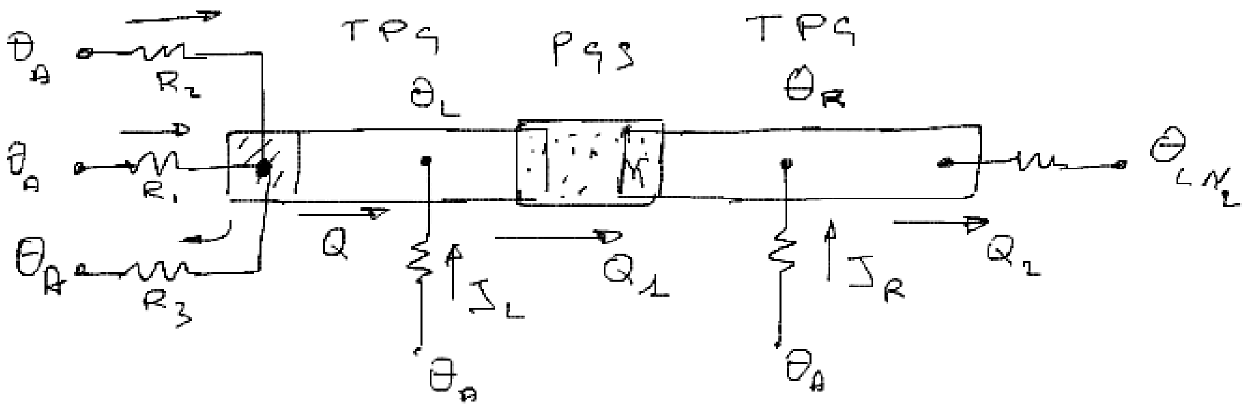

Fig.3 – Schematic of heat flow through the sample. The variables in the schematic are defined as:

Θ_A = room temperature
Θ_L = TPG left section average temperature
Q = Fraction of the H1 power entering the sample
R_1 = Thermal resistance of the H1's copper wires
R_2 = Thermal resistance of the 8 manganin wires to read the two RTDs on the top and bottom faces of the H1 heater
J_L = Power reaching the left section by radiation

Θ_{H1} = Heater H1 temperature
Θ_{LN2} = TPG right section temperature
Q_1 = Fraction of H1 power flowing through TPG left section
Q_2 = Fraction of H1 power flowing through TPG right section
R_3 = Thermal resistance of the pink insulation under the heater in series with the G10 plate (Figure 1).
J_R = Power reaching the right section by radiation

Equation (1) shows that the heat flow Q at the left TPG is the sum of the effects from the heater H1 and the thermal resistance R_p. R_p is the sum of the thermal resistances from the copper wires of the heater H1 (R_1), from the eight manganin wires connected to the two RTDs on the top and the bottom of the H_1 heater (R_2), and from the pink insulation under the heater in series with the G10 plate (R_3). R_p is the sum of all thermal resistances that sit in parallel ($\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$).

$$\frac{\theta_A - \theta_{H_1}}{R_p} + H_1 = Q \qquad (1)$$

Differentiating equation (1), the fraction of the H1 power step entering the sample is shown in equation (2):

$$\Delta Q = \Delta H_1 - \frac{\Delta \theta_{H_1}}{R_p} = \Delta H_1 \left(1 - \frac{\Delta \theta_{H_1} / R_p}{\Delta H_1} \right) \qquad (2)$$

The thermal resistance of the two copper (400 W/mK) wires one foot long and with a 0.010" diameter is R_1 = 7000 K/W. The contribution of the eight manganin (20 W/mK) wires, with 0.008" diameter, thermally anchored on the G10 plate is $R_2 \approx 10^4 K/W$. The pink foam piece under the heater (Figure 1) is made of Foamular 150 (Owens Corning) and the thermal conductivity @ 24C is 0.029 W/mK. The thickness is 0.5" and its contribution to the thermal resistance R_3 is about 1000 K/W. Adding the G10 (0.3 W/mK @120K and 0.45 W/mK at room temperature) contribution of about 2000 K/W we have $R_3 \approx 3000 K/W$. So the parallel resistance R_p is 1000 - 2000 K/W.

The thermometer in the position 10 of the Figure 5 and 6 (BTeV document 1650) is attached to the TPG under the heater; the change of the heater temperature is about 21 C when the H1 heater changes of 1 W.

The power leak through R_p is

$$\frac{\Delta \theta_{H_1}}{R_p} = \frac{21\ K}{(1000 \div 2000)\frac{K}{W}} \cong (10 \div 20)\ mW$$

$$\frac{\Delta \theta_{H_1}/R_p}{\Delta H_1} = \frac{10 \div 20\ mW}{1\ W} = (1 \div 2) \times 10^{-2}$$

Thus, from equation (2), the fraction of the H1 power step entering the TPG is

$$\Delta Q = 0.98\ \Delta H_1 \qquad (3)$$

Going back to the heat flow schematic in Figure 3, we have the total heat flow through the PGS as the sum of the fraction of power coming from the heater H1 and the power due to radiation:

$$Q_1 = Q + J_L = Q + \varepsilon A \sigma \left(\theta_a^4 - \theta_L^4\right) \qquad (4)$$

where ε=0.9 is the surface emissivity, σ=5.67 $10^{-8}\ W/m^2K^4$ is the Stefan's constant and $A \cong 44\,cm^2$ is cross sectional area between the upper and lower surface of half sample. The fraction of the power step from heater H1 reaching the right TPG sample because of this radiation coupling is

$$\Delta Q_1 = \Delta Q - 4 \varepsilon A \sigma \theta_L^3\ \Delta \theta_L \qquad (5)$$

The thermal resistance of this coupling depends strongly on the average sample temperature (Θ_L^3)

$$\frac{1}{4 \varepsilon A \sigma \theta_L^3} \cong \begin{cases} 1100\ \frac{K}{W} & \text{for } \theta_L = 100K \\ 330\ \frac{K}{W} & \text{for } \theta_L = 150K \\ 140\ \frac{K}{W} & \text{for } \theta_L = 200K \\ 70\ \frac{K}{W} & \text{for } \theta_L = 250K \end{cases}$$

Note that this resistance calculation has been done only to show the importance of radiation coupling. However, we will not use its value to obtain the sample thermal conductivity. Again from the data in the file *TPG_PGS Temp.Distribution (H2=0W).xls* we have that the average temperature change of the left TPG sample is 20°C when the heater power changes 1 W. So the heat power leaking by this radiation coupling is

$$4 \varepsilon A \sigma \theta_L^3 \Delta \theta_L = \begin{cases} 1.8 \times 10^{-2}\ W & \text{for } \theta_L = 100\ K \\ 6.1 \times 10^{-2}\ W & " \quad \theta_L = 150\ K \\ 14 \times 10^{-2}\ W & \theta_L = 200\ K \\ 28 \times 10^{-2}\ W & \theta_L = 250\ K \end{cases}$$

For the higher values of the heater power, it can reach the 30% of the H1 power step (1W). The power leaking from the right TPG sample is much lower because both its temperature Θ_R and its temperature change are lower.

Assume that the relationship between the power Q_2 reaching the cold end of the sample and the heater power H_1 is as shown in equation (6):

$$Q_2 = 0.98\, H_1 \left(1 - \frac{C}{B} H_1\right) \qquad (6)$$

Then, the relationship between the RTD201 temperature and this power flow Q_2 becomes linear as shown in equation (7):

$$RTD201 = A + \frac{B}{0.98} Q_2 \qquad (7)$$

This is expected if the thermal resistance of the link between the RTD201 and the LN2 sink does not change with temperature. B and C are the coefficients of the second order polynomial fit of the RTD201 temperature. The relative algebra is reported in the Appendix A.

To calculate the thermal conductivity we make the difference between the temperature gradients obtained with two heat flows through the sample (BTeV 1650). Now we use the heat flow, reaching the sample cold end, Q_2 to calculate the thermal conductivity of the right TPG sample.

$$Q_2'' = k S \left(\frac{\partial \theta}{\partial x}\right)'' \qquad Q_2' = k S \left(\frac{\partial \theta}{\partial x}\right)'$$

$$k = \frac{Q_2'' - Q_2'}{S\left[\left(\frac{\partial \theta}{\partial x}\right)'' - \left(\frac{\partial \theta}{\partial x}\right)'\right]} \qquad (8)$$

Using equation (6) we calculate the difference between the heat flows reaching the cold end with two different heater powers

$$Q_2'' - Q_2' = 0.98\left(H_1'' - H_1'\right)\left(1 - \frac{C}{B}\left(H_1'' + H_1'\right)\right) \quad (9)$$

Note that the term $0.98(H_1''-H_1')$ is the fraction (3) of the H1 power change entering the sample and the term $(1-\frac{C}{B}(H_1^{"}+H_1^{'}))$ accounts for the heat loss from the left TPG section by the radiation coupling. Knowing that the majority of the heat loss by radiation leaves the left TPG section, we use the (9) to calculate the right TPG and PGS conductivity and

$$\left.\begin{matrix}\text{HEAT FLOW}\\ \text{DIFFERENCE}\\ \text{TO CALCULATE}\\ \text{THE THERMAL}\\ \text{CONDUCTIVITY}\\ \text{OF THE LEFT}\\ \text{TPG SAMPLE}\end{matrix}\right\} = 0.98\left(H_1'' - H_1'\right)\left(1 - \frac{1}{2}\frac{C}{B}\left(H_1'' + H_1'\right)\right) \quad (10)$$

to calculate the left TPG conductivity.

Data Analysis Method

1. Measure the sample temperature distribution with different H1 heater powers.
2. Obtain the temperature gradients in the left TPG, the PGS and the right TPG sample sections.
3. Obtain the coefficient C and B from second order polynomial fit of the RTD201 temperature vs the H1 heater power (Fig.2 and (3A)).
4. Calculate the difference between the heat flows crossing the PGS and the right TPG using equation (9) and the heat flows crossing the left TPG section using equation (10).
5. Making the difference between the corresponding temperature gradients, calculate the thermal conductivity of the sample at the average of the temperatures used to calculate the appropriate gradients.

The conductivities obtained by this analysis method are reported in the following Figures 4 and 5.

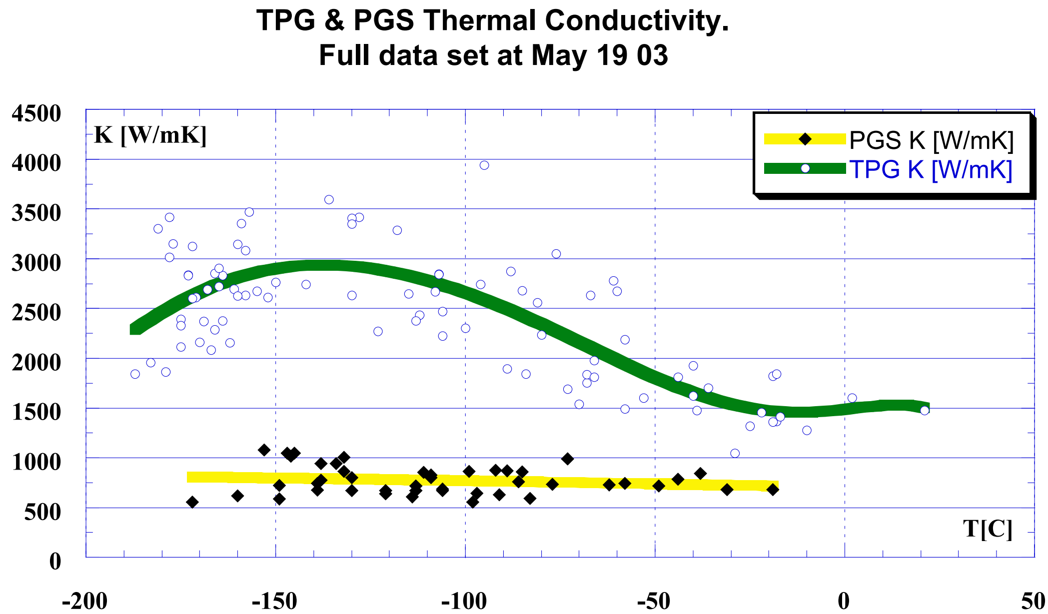

Figure 4 – The TPG and PGS thermal conductivities using the full data set as of May 19, 2003.

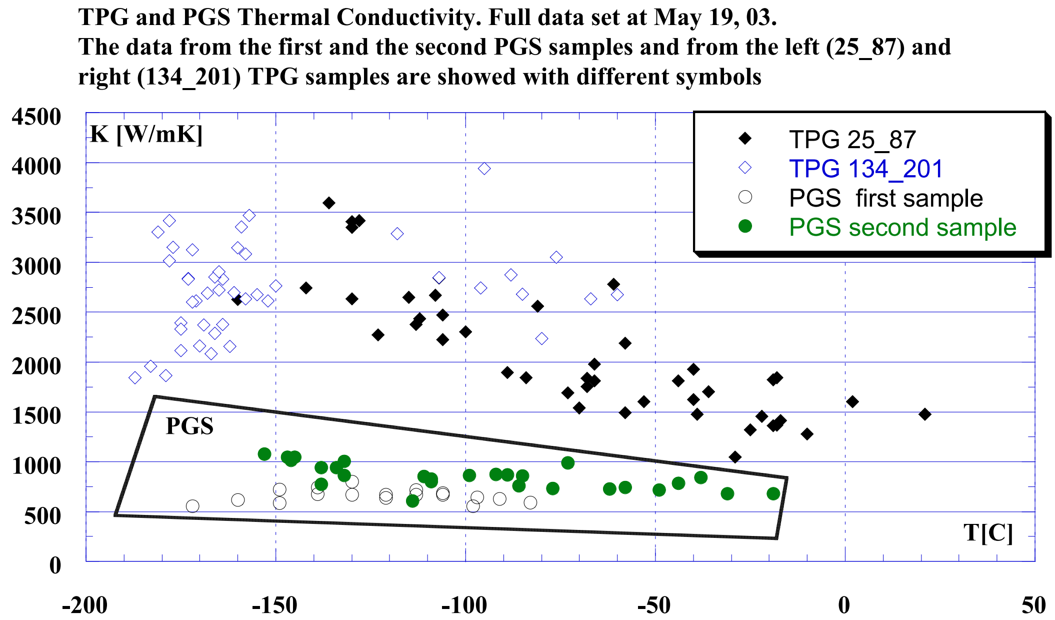

Figure 5 – The thermal conductivities of each material shown according to TPG section and PGS samples. The first PGS sample was accidentally broken at the joint and had to be reglued. The second PGS sample is the new sample after being fixed.

Conclusion

The differential method of analysis takes into account parameters that do not change when the heater power changes, such as emissivity or the vessel temperature. Knowing the emissivity of all the surfaces involved in this heat exchange and the vessel temperature, it is also possible to solve the problem by modeling of all the heat flow paths using finite element analysis and finding the conductivity distribution that gives the minimum squared differences between the calculated and measured temperatures.

The data shows that the thermal conductivity of TPG is affected by cold temperature while the thermal conductivity of PGS remains unchanged.

Appendix A

We show that choosing

$$Q_2 = 0.38\, H_1 \left(1 - \frac{C}{B} H_1\right) \qquad (1A)$$

the relationship between the RTD201 temperature and the heat flow Q_2 becomes linear. From (1A) we have

$$\Rightarrow H_1 = \frac{1}{2}\left[\frac{B}{C} \pm \sqrt{\left(\frac{B}{C}\right)^2 - \frac{4\, Q_2\, B}{0.38\, C}}\right] \qquad (2A)$$

Substituting the (2A) in the equation (3A) that is obtained from the fit of the RTD201 data,

$$RTD201 = A + B H_1 - C H_1^2 \qquad (3A)$$

we have the linear dependence of the RTD201 temperature from the heat flow reaching the cold end of the sample.

$$RTD201 = A + \frac{B}{0.38} Q_2 \qquad (7)$$